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2006 J. Phys. A: Math. Gen. 39 10549

(http://iopscience.iop.org/0305-4470/39/33/021)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 10549–10564 doi:10.1088/0305-4470/39/33/021

Interactions between a massless tensor field with the mixed symmetry of the Riemann tensor and a massless vector field

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Received 10 April 2006, in final form 3 July 2006
Published 2 August 2006
Online at stacks.iop.org/JPhysA/39/10549

Abstract

Consistent couplings between a massless tensor field with the mixed symmetry of the Riemann tensor and a massless vector field are analysed in the framework of Lagrangian BRST cohomology. Under the assumptions on smoothness, locality, Lorentz covariance and Poincaré invariance of the deformations, combined with the requirement that the interacting Lagrangian is at most second-order derivative, it is proved that there are no consistent cross-interactions between a single massless tensor field with the mixed symmetry of the Riemann tensor and one massless vector field.

PACS number: 11.10.Ef

1. Introduction

Mixed symmetry-type tensor fields [1–7] are involved in many physically interesting theories, such as superstrings, supergravities, or supersymmetric high spin theories. The study of gauge theories with mixed symmetry-type tensor fields raised several issues, such as the dual formulation of field theories of spin two or higher [8–15], the impossibility of consistent interactions in the dual formulation of linearized gravity [16], or a Lagrangian first-order approach [12, 17, 18] to some classes of free massless mixed symmetry-type tensor gauge fields, suggestively resembling to the tetrad formalism of general relativity. One of the most important aspects related to this type of gauge models is the analysis of their consistent interactions, among themselves as well as with higher spin gauge theories [19–28]. The best approach to this matter is the cohomological one, based on the deformation of the solution to the master equation [29]. The aim of our paper is to investigate the manifestly covariant consistent interactions between a single, free, massless tensor gauge field $t_{\mu\nu|\alpha\beta}$ with the mixed symmetry of the Riemann tensor and a massless vector field.

Our procedure relies on the deformation of the solution to the master equation by means of local BRST cohomology. For each situation, we initially determine the associated free antifield-BRST symmetry s, which splits as the sum between the Koszul–Tate differential and the exterior longitudinal derivative only, $s = \delta + \gamma$. Then, we solve the basic equations of the deformation procedure. Under the supplementary assumptions on smoothness, locality, Lorentz covariance and Poincaré invariance of the deformations as well as on the maximum derivative order of the interacting Lagrangian being equal to 2, we prove that there are no consistent cross-interactions between the tensor field with the mixed symmetry of the Riemann tensor and the massless vector field.

The paper is organized in four sections. Section 2 is focused on the presentation of the free model under study and on the construction of the associated BRST differential. In section 3 we briefly review the antifield-BRST deformation procedure. Section 4 analyses the consistent couplings between the tensor field with the mixed symmetry of the Riemann tensor and the massless vector field with the help of the local BRST cohomology of the free model. Section 5 ends the paper with some conclusions.

2. Free model: free BRST symmetry

The starting point is given by the free Lagrangian action

$$\begin{aligned} S_0[t_{\mu\nu|\alpha\beta}, A_\mu] = \int \mathrm{d}^D x \left[\frac{1}{8}(\partial^\lambda t^{\mu\nu|\alpha\beta})(\partial_\lambda t_{\mu\nu|\alpha\beta}) - (\partial_\mu t^{\mu\nu|\alpha\beta})(\partial_\beta t_{\nu\alpha}) \right.\\ - \frac{1}{2}(\partial_\mu t^{\mu\nu|\alpha\beta})(\partial^\lambda t_{\lambda\nu|\alpha\beta}) - \frac{1}{2}(\partial^\lambda t^{\nu\beta})(\partial_\lambda t_{\nu\beta})\\ + (\partial_\nu t^{\nu\beta})(\partial^\lambda t_{\lambda\beta}) - \frac{1}{2}(\partial_\nu t^{\nu\beta})(\partial_\beta t)\\ \left. + \frac{1}{8}(\partial^\lambda t)(\partial_\lambda t) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right] \equiv S_0^{\mathrm{t}}[t_{\mu\nu|\alpha\beta}] + S_0^{\mathrm{A}}[A_\mu] \qquad (1) \end{aligned}$$

in a Minkowski-flat spacetime of dimension $D \geqslant 5$, endowed with a metric tensor of 'mostly plus' signature $\sigma_{\mu\nu} = \sigma^{\mu\nu} = (-++++\cdots)$. The massless tensor field $t_{\mu\nu|\alpha\beta}$ of rank four has the mixed symmetry of the Riemann tensor and hence transforms according to an irreducible representation of $GL(D,\mathbb{R})$ corresponding to a rectangular Young diagram with two columns and two rows. Thus, it is separately antisymmetric in the pairs $\{\mu,\nu\}$ and $\{\alpha,\beta\}$, is symmetric under the interchange of these pairs ($\{\mu,\nu\} \longleftrightarrow \{\alpha,\beta\}$) and satisfies the identity $t_{[\mu\nu|\alpha]\beta} \equiv 0$ associated with the above diagram, which we will refer to as the Bianchi I identity. Here and in the following the symbol $[\mu\nu\cdots]$ signifies complete antisymmetry with respect to the (Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. (For instance, we have that $t_{[\mu\nu|\alpha]\beta} = t_{\mu\nu|\alpha\beta} + t_{\nu\alpha|\mu\beta} + t_{\alpha\mu|\nu\beta}$.) The notation $t_{\nu\beta}$ signifies the simple trace of the original tensor field, $t_{\nu\beta} = \sigma^{\mu\alpha}t_{\mu\nu|\alpha\beta}$, which is symmetric, $t_{\nu\beta} = t_{\beta\nu}$, while t denotes its double trace, $t = \sigma^{\nu\beta}t_{\nu\beta} \equiv t^{\mu\nu}{}_{|\mu\nu}$, which is a scalar. A generating set of gauge transformations for action (1) reads as

$$\delta_\epsilon t_{\mu\nu|\alpha\beta} = \partial_\mu \epsilon_{\alpha\beta|\nu} - \partial_\nu \epsilon_{\alpha\beta|\mu} + \partial_\alpha \epsilon_{\mu\nu|\beta} - \partial_\beta \epsilon_{\mu\nu|\alpha}, \qquad \delta_\epsilon A_\mu = \partial_\mu \epsilon, \qquad (2)$$

with the bosonic gauge parameters $\epsilon_{\mu\nu|\alpha}$ transforming according to an irreducible representation of $GL(D,\mathbb{R})$ corresponding to a three-cell Young diagram with two columns and two rows (also known as a hook diagram), being therefore antisymmetric in the pair $\mu\nu$ and satisfying the identity $\epsilon_{[\mu\nu|\alpha]} \equiv 0$. The last identity is required in order to ensure that the gauge transformations (2) check the same Bianchi I identity like the fields themselves, namely,

$\delta_\epsilon t_{[\mu\nu|\alpha]\beta} \equiv 0$. The above generating set of gauge transformations is Abelian and off-shell, first-stage reducible since if we make the transformation

$$\epsilon_{\mu\nu|\alpha} = 2\partial_\alpha\theta_{\mu\nu} - \partial_{[\mu}\theta_{\nu]\alpha}, \tag{3}$$

with $\theta_{\mu\nu}$ being an arbitrary antisymmetric tensor ($\theta_{\mu\nu} = -\theta_{\nu\mu}$), then the gauge transformations of the tensor field identically vanish, $\delta_{\epsilon(\theta)} t_{\mu\nu|\alpha\beta} \equiv 0$.

In agreement with the general setting of the antibracket–antifield formalism, the construction of the BRST symmetry for the free theory under consideration starts with the identification of the BRST algebra on which the BRST differential s acts. The generators of the BRST algebra are of two kinds: fields/ghosts and antifields. The ghost spectrum for the model under study comprises the fermionic ghosts $\eta_{\alpha\beta|\mu}$ and η associated with the gauge parameters $\epsilon_{\alpha\beta|\mu}$ and ϵ from (2) as well as the bosonic ghosts for ghosts $C_{\mu\nu}$ due to the first-stage reducibility parameters $\theta_{\mu\nu}$ in (3). In order to make compatible the behaviour of $\epsilon_{\alpha\beta|\mu}$ and $\theta_{\mu\nu}$ with that of the corresponding ghosts, we assume that $\eta_{\alpha\beta|\mu}$ satisfies the properties $\eta_{\mu\nu|\alpha} = -\eta_{\nu\mu|\alpha}$, $\eta_{[\mu\nu|\alpha]} \equiv 0$ and that $C_{\mu\nu}$ is antisymmetric. The antifield spectrum is organized into the antifields $t^{*\mu\nu|\alpha\beta}$ and $A^{*\mu}$ of the original tensor fields and those of the ghosts, $\eta^{*\mu\nu|\alpha}$, η^* and $C^{*\mu\nu}$, of statistics opposite to that of the associated fields/ghosts. It is understood that $t^{*\mu\nu|\alpha\beta}$ is subject to the conditions

$$t^{*\mu\nu|\alpha\beta} = -t^{*\nu\mu|\alpha\beta} = -t^{*\mu\nu|\beta\alpha} = t^{*\alpha\beta|\mu\nu}, \qquad t^{*[\mu\nu|\alpha]\beta} \equiv 0 \tag{4}$$

and, along the same line, it is required that

$$\eta^{*\mu\nu|\alpha} = -\eta^{*\nu\mu|\alpha}, \qquad \eta^{*[\mu\nu|\alpha]} \equiv 0, \qquad C^{*\mu\nu} = -C^{*\nu\mu}. \tag{5}$$

We will denote the simple and double traces of $t^{*\mu\nu|\alpha\beta}$ by

$$t^{*\nu\beta} = \sigma_{\mu\alpha} t^{*\mu\nu|\alpha\beta}, \qquad t^* = \sigma_{\nu\beta} t^{*\nu\beta}, \tag{6}$$

such that $t^{*\nu\beta}$ is a symmetric and t^* is a scalar.

As both the gauge generators and reducibility functions for this model are field independent, it follows that the associated BRST differential ($s^2 = 0$) splits into

$$s = \delta + \gamma, \tag{7}$$

where δ represents the Koszul–Tate differential ($\delta^2 = 0$), graded by the antighost number agh ($\mathrm{agh}(\delta) = -1$), and γ stands for the exterior derivative along the gauge orbits. It turns out to be a true differential ($\gamma^2 = 0$) that anticommutes with δ ($\delta\gamma + \gamma\delta = 0$), whose degree is named pure ghost number pgh ($\mathrm{pgh}(\gamma) = 1$). These two degrees do not interfere ($\mathrm{agh}(\gamma) = 0$, $\mathrm{pgh}(\delta) = 0$). The overall degree that grades the BRST differential is known as the ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(s) = \mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = 1$. According to the standard rules of the BRST method, the corresponding degrees of the generators from the BRST complex are valued as

$$\mathrm{pgh}(t_{\mu\nu|\alpha\beta}) = 0 = \mathrm{pgh}(A_\mu) \tag{8}$$

$$\mathrm{pgh}(\eta_{\mu\nu|\alpha}) = 1 = \mathrm{pgh}(\eta), \qquad \mathrm{pgh}(C_{\mu\nu}) = 2 \tag{9}$$

$$\mathrm{pgh}(t^{*\mu\nu|\alpha\beta}) = \mathrm{pgh}(A^{*\mu}) = \mathrm{pgh}(\eta^{*\mu\nu|\alpha}) = \mathrm{pgh}(\eta^*) = \mathrm{pgh}(C^{*\mu\nu}) = 0 \tag{10}$$

$$\mathrm{agh}(t_{\mu\nu|\alpha\beta}) = \mathrm{agh}(A_\mu) = \mathrm{agh}(\eta_{\mu\nu|\alpha}) = \mathrm{agh}(\eta) = \mathrm{agh}(C_{\mu\nu}) = 0 \tag{11}$$

$$\mathrm{agh}(t^{*\mu\nu|\alpha\beta}) = 1 = \mathrm{agh}(A^{*\mu}) \tag{12}$$

$$\mathrm{agh}(\eta^{*\mu\nu|\alpha}) = 2 = \mathrm{agh}(\eta^*), \qquad \mathrm{agh}(C^{*\mu\nu}) = 3, \tag{13}$$

and the actions of δ and γ on them are given by

$$\gamma t_{\mu\nu|\alpha\beta} = \partial_\mu \eta_{\alpha\beta|\nu} - \partial_\nu \eta_{\alpha\beta|\mu} + \partial_\alpha \eta_{\mu\nu|\beta} - \partial_\beta \eta_{\mu\nu|\alpha} \tag{14}$$

$$\gamma \eta_{\mu\nu|\alpha} = 2\partial_\alpha C_{\mu\nu} - \partial_{[\mu} C_{\nu]\alpha}, \qquad \gamma C_{\mu\nu} = 0 \tag{15}$$

$$\gamma t^{*\mu\nu|\alpha\beta} = \gamma \eta^{*\mu\nu|\alpha} = \gamma C^{*\mu\nu} = 0 \tag{16}$$

$$\delta t_{\mu\nu|\alpha\beta} = \delta \eta_{\mu\nu|\alpha} = \delta C_{\mu\nu} = 0 \tag{17}$$

$$\delta t^{*\mu\nu|\alpha\beta} = \tfrac{1}{4} T^{\mu\nu|\alpha\beta}, \qquad \delta \eta^{*\alpha\beta|\nu} = -4\partial_\mu t^{*\mu\nu|\alpha\beta}, \qquad \delta C^{*\mu\nu} = 3\partial_\alpha \eta^{*\mu\nu|\alpha} \tag{18}$$

$$\gamma A_\mu = \partial_\mu \eta, \qquad \gamma \eta = 0, \qquad \gamma A^{*\mu} = 0, \qquad \gamma \eta^* = 0 \tag{19}$$

$$\delta A_\mu = 0, \qquad \delta \eta = 0, \qquad \delta A^{*\mu} = -\partial_\nu F^{\mu\nu}, \qquad \delta \eta^* = -\partial_\mu A^{*\mu}, \tag{20}$$

with $T_{\mu\nu|\alpha\beta}$ of the form

$$\begin{aligned} T_{\mu\nu|\alpha\beta} = {} & \Box t_{\mu\nu|\alpha\beta} + \partial^\rho (\partial_\mu t_{\alpha\beta|\nu\rho} - \partial_\nu t_{\alpha\beta|\mu\rho} + \partial_\alpha t_{\mu\nu|\beta\rho} - \partial_\beta t_{\mu\nu|\alpha\rho}) \\ & + (\partial_\mu \partial_\alpha t_{\beta\nu} - \partial_\mu \partial_\beta t_{\alpha\nu} - \partial_\nu \partial_\alpha t_{\beta\mu} + \partial_\nu \partial_\beta t_{\alpha\mu}) \\ & - \tfrac{1}{2} \partial^\lambda \partial^\rho (\sigma_{\mu\alpha} (t_{\lambda\beta|\nu\rho} + t_{\lambda\nu|\beta\rho}) - \sigma_{\mu\beta} (t_{\lambda\alpha|\nu\rho} + t_{\lambda\nu|\alpha\rho}) \\ & - \sigma_{\nu\alpha} (t_{\lambda\beta|\mu\rho} + t_{\lambda\mu|\beta\rho}) + \sigma_{\nu\beta} (t_{\lambda\alpha|\mu\rho} + t_{\lambda\mu|\alpha\rho})) \\ & - \Box (\sigma_{\mu\alpha} t_{\beta\nu} - \sigma_{\mu\beta} t_{\alpha\nu} - \sigma_{\nu\alpha} t_{\beta\mu} + \sigma_{\nu\beta} t_{\alpha\mu}) \\ & + \partial^\rho (\sigma_{\mu\alpha} (\partial_\beta t_{\nu\rho} + \partial_\nu t_{\beta\rho}) - \sigma_{\mu\beta} (\partial_\alpha t_{\nu\rho} + \partial_\nu t_{\alpha\rho}) \\ & - \sigma_{\nu\alpha} (\partial_\beta t_{\mu\rho} + \partial_\mu t_{\beta\rho}) + \sigma_{\nu\beta} (\partial_\alpha t_{\mu\rho} + \partial_\mu t_{\alpha\rho})) \\ & - \tfrac{1}{2} (\sigma_{\mu\alpha} \partial_\beta \partial_\nu - \sigma_{\mu\beta} \partial_\alpha \partial_\nu - \sigma_{\nu\alpha} \partial_\beta \partial_\mu + \sigma_{\nu\beta} \partial_\alpha \partial_\mu) t \\ & - (\sigma_{\mu\alpha} \sigma_{\nu\beta} - \sigma_{\mu\beta} \sigma_{\nu\alpha}) \big(\partial^\lambda \partial^\rho t_{\lambda\rho} - \tfrac{1}{2} \Box t \big). \end{aligned} \tag{21}$$

Both δ and γ (and implicitly s) were taken to act like right derivations.

The antifield-BRST differential is known to admit a canonical action in a structure named antibracket and defined by decreeing the fields/ghosts conjugated with the corresponding antifields, $s\cdot = (\cdot, S)$, where $(,)$ signifies the antibracket and S denotes the canonical generator of the BRST symmetry. It is a bosonic functional of ghost number 0 involving both the field/ghost and antifield spectra, which obeys the classical master equation

$$(S, S) = 0. \tag{22}$$

The classical master equation is equivalent with the second-order nilpotency of s, $s^2 = 0$, while its solution encodes the entire gauge structure of the associated theory. Taking into account the formulae (14)–(20) as well as the actions of δ and γ in canonical form, we find that the complete solution to the master equation for the model under study reads as

$$\begin{aligned} S = S_0[t_{\mu\nu|\alpha\beta}, A_\mu] + \int \mathrm{d}^D x [& t^{*\mu\nu|\alpha\beta} (\partial_\mu \eta_{\alpha\beta|\nu} - \partial_\nu \eta_{\alpha\beta|\mu} + \partial_\alpha \eta_{\mu\nu|\beta} \\ & - \partial_\beta \eta_{\mu\nu|\alpha}) + \eta^{*\mu\nu|\alpha} (2\partial_\alpha C_{\mu\nu} - \partial_{[\mu} C_{\nu]\alpha}) + A^{*\mu} \partial_\mu \eta]. \end{aligned} \tag{23}$$

The main ingredients of the antifield-BRST symmetry derived in this section will be useful in the following at the analysis of consistent interactions that can be added to action (1) without changing its number of independent gauge symmetries.

3. Brief review of the antifield-BRST deformation procedure

There are three main types of consistent interactions that can be added to a given gauge theory: the first type deforms only the Lagrangian action, but not its gauge transformations; the second kind modifies both the action and its transformations, but not the gauge algebra and the third, and certainly most interesting category, changes everything, namely, the action, its gauge symmetries and the accompanying algebra.

The reformulation of the problem of consistent deformations of a given action and of its gauge symmetries in the antifield-BRST setting (for a comprehensive review, see [30]) is based on the observation that if a deformation of the classical theory can be consistently constructed, then the solution to the master equation for the initial theory can be deformed into

$$\bar{S} = S + gS_1 + g^2S_2 + O(g^3), \qquad \varepsilon(\bar{S}) = 0, \quad \mathrm{gh}(\bar{S}) = 0 \tag{24}$$

such that

$$(\bar{S}, \bar{S}) = 0. \tag{25}$$

Here and in the following $\varepsilon(F)$ denotes the Grassmann parity of F. The projection of (25) on the various powers in the coupling constant induces the following tower of equations:

$$g^0 : (S, S) = 0 \tag{26}$$

$$g^1 : (S_1, S) = 0 \tag{27}$$

$$g^2 : \tfrac{1}{2}(S_1, S_1) + (S_2, S) = 0 \\ \vdots \tag{28}$$

The first equation is satisfied by hypothesis. The second one governs the first-order deformation of the solution to the master equation (S_1) and it shows that S_1 is a BRST co-cycle, $sS_1 = 0$. This means that S_1 pertains to the ghost number 0 cohomological space of s, $H^0(s)$, which is generically non-empty due to its isomorphism to the space of physical observables of the free theory. The remaining equations are responsible for the higher order deformations of the solution to the master equation. No obstructions arise in finding solutions to them as long as no further restrictions, such as spacetime locality, are imposed. Obviously, only nontrivial first-order deformations should be considered, since trivial ones ($S_1 = sB$) lead to trivial deformations of the initial theory and can be eliminated by convenient redefinitions of the fields. Ignoring the trivial deformations, it follows that S_1 is a nontrivial BRST observable, $S_1 \in H^0(s)$. Once that the deformation equations (27)–(28), etc have been solved by means of specific cohomological techniques, from the consistent nontrivial deformed solution to the master equation we can extract all the information on the gauge structure of the accompanying interacting theory.

4. First-order deformation

The purpose of our paper is to study the consistent interactions that can be added to the free action (1) by means of solving the main deformation equations, namely, (27)–(28), etc. For obvious reasons, we consider only smooth, local, Lorentz-covariant and Poincaré-invariant deformations. If we make the notation $S_1 = \int \mathrm{d}^D x\, a$, with a a local function, then the local form of (27), which we have seen that controls the first-order deformation of the solution to the master equation, becomes

$$sa = \partial_\mu m^\mu, \qquad \mathrm{gh}(a) = 0, \qquad \varepsilon(a) = 0 \tag{29}$$

for some m^μ, and it shows that the nonintegrated density of the first-order deformation pertains to the local cohomology of s at ghost number 0, $a \in H^0(s|\mathrm{d})$, where d denotes the exterior spacetime differential. In order to analyse the above equation, we develop a according to the antighost number

$$a = \sum_{k=0}^{I} a_k, \qquad \mathrm{agh}(a_k) = k, \qquad \mathrm{gh}(a_k) = 0, \qquad \varepsilon(a_k) = 0 \tag{30}$$

and assume, without loss of generality, that a stops at some finite value I of the antighost number. (This can be shown, for instance, like in [31] (section 3) or [32], under the sole assumption that the interacting Lagrangian at the first order in the coupling constant, a_0, has a finite, but otherwise arbitrary derivative order.) By taking into account the decomposition (7) of the BRST differential, (29) is equivalent to a tower of local equations, corresponding to the various decreasing values of the antighost number:

$$\gamma a_I = \partial_\mu \overset{(I)}{m}{}^{\mu} \tag{31}$$

$$\delta a_I + \gamma a_{I-1} = \partial_\mu \overset{(I-1)}{m}{}^{\mu} \tag{32}$$

$$\delta a_k + \gamma a_{k-1} = \partial_\mu \overset{(k-1)}{m}{}^{\mu}, \qquad I-1 \geqslant k \geqslant 1, \tag{33}$$

where $\left(\overset{(k)}{m}{}^{\mu}\right)_{k=\overline{0,I}}$ are some local currents, with $\mathrm{agh}\left(\overset{(k)}{m}{}^{\mu}\right) = k$. It can be proved that one can replace (31) at strictly positive antighost numbers with

$$\gamma a_I = 0, \qquad I > 0. \tag{34}$$

(The fact that it is possible to replace (31) with (34) can be applied like in the proof of corollary 3 from [33], with the precaution to include in an appropriate manner the dependence on the vector field BRST sector.) In conclusion, under the assumption that $I > 0$, the representative of highest antighost number from the nonintegrated density of the first-order deformation can always be taken to be γ-closed, such that (29) associated with the local form of the first-order deformation is completely equivalent to the tower (34) and (32)–(33).

Before proceeding to the analysis of the solutions to the first-order deformation equation, we briefly comment on the uniqueness and triviality of such solutions. Due to the second-order nilpotency of γ ($\gamma^2 = 0$), the solution to (34) is clearly unique up to γ-exact contributions,

$$a_I \rightarrow a_I + \gamma b_I, \qquad \mathrm{agh}(b_I) = I, \qquad \mathrm{pgh}(b_I) = I-1, \qquad \varepsilon(b_I) = 1. \tag{35}$$

Meanwhile, if it turns out that a_I reduces to γ-exact terms only, $a_I = \gamma b_I$, then it can be made to vanish, $a_I = 0$. In other words, the nontriviality of the first-order deformation a is translated at its highest antighost number component into the requirement that

$$a_I \in H^I(\gamma), \tag{36}$$

where $H^I(\gamma)$ denotes the cohomology of the exterior longitudinal derivative γ at pure ghost number equal to I. At the same time, the general condition on the nonintegrated density of the first-order deformation to be in a nontrivial cohomological class of $H^0(s|\mathrm{d})$ shows on the one hand that the solution to (29) is unique up to s-exact pieces plus total divergences:

$$\begin{aligned} &a \rightarrow a + sb + \partial_\mu n^\mu, \qquad &&\mathrm{gh}(b) = -1, \qquad &&\varepsilon(b) = 1, \\ &\mathrm{gh}(n^\mu) = 0, &&\varepsilon(n^\mu) = 0, \end{aligned} \tag{37}$$

and on the other hand that if the general solution to (29) is found to be completely trivial, $a = sb + \partial_\mu n^\mu$, then it can be made to vanish, $a = 0$.

4.1. Basic cohomologies

In the light of the above discussion, we pass to the investigation of the solutions to (34) and (32)–(33). We have seen that a_I belongs to the cohomology of the exterior longitudinal derivative (see the formula (36)), such that we need to compute $H(\gamma)$ in order to construct the component of highest antighost number from the first-order deformation. This matter is solved with the help of the definitions (14)–(16) and (19).

In order to determine the cohomology $H(\gamma)$, we split the differential γ into two pieces:

$$\gamma = \gamma_t + \gamma_A, \tag{38}$$

where γ_t acts nontrivially only on the fields/ghosts from the $t_{\mu\nu|\alpha\beta}$ sector and γ_A does the same thing, but with respect to the vector field sector. From the above splitting it follows that the nilpotency of γ is equivalent to the nilpotency and anticommutativity of its components

$$(\gamma_t)^2 = 0 = (\gamma_A)^2, \qquad \gamma_t\gamma_A + \gamma_A\gamma_t = 0. \tag{39}$$

Kunneth's formula then ensures the isomorphism

$$H(\gamma) = H(\gamma_t) \otimes H(\gamma_A). \tag{40}$$

Thus, we can state that $H(\gamma)$ is generated [36] on the one hand by $\chi^{*\Delta}$, $F_{\mu\nu}$ and $F_{\mu\nu\lambda|\alpha\beta\gamma}$ as well as by their spacetime derivatives and on the other hand by the ghosts $C_{\mu\nu}$, $\partial_{[\mu}C_{\nu]\alpha}$ and η, where $\chi^{*\Delta}$ is a collective notation for all the antifields:

$$\chi^{*\Delta} = \{t^{*\mu\nu|\alpha\beta}, A^{*\mu}, \eta^{*\mu\nu|\alpha}, \eta^{*}, C^{*\mu\nu}\}, \tag{41}$$

while

$$\begin{aligned} F_{\mu\nu\lambda|\alpha\beta\gamma} = {} & \partial_\lambda\partial_\gamma t_{\mu\nu|\alpha\beta} + \partial_\mu\partial_\gamma t_{\nu\lambda|\alpha\beta} + \partial_\nu\partial_\gamma t_{\lambda\mu|\alpha\beta} + \partial_\lambda\partial_\alpha t_{\mu\nu|\beta\gamma} + \partial_\mu\partial_\alpha t_{\nu\lambda|\beta\gamma} \\ & + \partial_\nu\partial_\alpha t_{\lambda\mu|\beta\gamma} + \partial_\lambda\partial_\beta t_{\mu\nu|\gamma\alpha} + \partial_\mu\partial_\beta t_{\nu\lambda|\gamma\alpha} + \partial_\nu\partial_\beta t_{\lambda\mu|\gamma\alpha} \end{aligned} \tag{42}$$

represent the components of the curvature tensor for $t_{\mu\nu|\alpha\beta}$ (the quantities with the minimum number of derivatives, invariant under the gauge transformations $\delta_\epsilon t_{\mu\nu|\alpha\beta}$ in (2)). (The quantity $\partial_{[\mu}C_{\nu]\alpha}$ carries a trivial component. Its nontrivial part is given by the completely antisymmetric expression $\partial_{[\mu}C_{\nu\alpha]}$, which differs from our representative by a γ-exact term.) So, the most general (and nontrivial), local solution to (34) can be written, up to γ-exact contributions, as

$$a_I = \alpha_I([F_{\mu\nu}], [F_{\mu\nu\lambda|\alpha\beta\gamma}], [\chi^{*\Delta}])\omega^I(C_{\mu\nu}, \partial_{[\mu}C_{\nu]\alpha}, \eta), \tag{43}$$

where the notation $f([q])$ means that f depends on q and its derivatives up to a finite order and ω^I denotes the elements of a basis in the space of polynomials with pure ghost number I in the corresponding ghosts and some of their first-order derivatives. The objects α_I (obviously nontrivial in $H^0(\gamma)$) were taken to have a bounded number of derivatives and therefore they are polynomials in the antifields $\chi^{*\Delta}$, in $F_{\mu\nu}$, in the curvature tensor $F_{\mu\nu\lambda|\alpha\beta\gamma}$ as well as in their derivatives. Due to the fact that these elements are γ-closed, they are called invariant polynomials. At zero antighost number, the invariant polynomials are polynomials in the curvature tensor $F_{\mu\nu\lambda|\alpha\beta\gamma}$, the field strength $F_{\mu\nu}$ and their derivatives.

Replacing solution (43) into (32), we remark that a necessary (but not sufficient) condition for the existence of (nontrivial) solutions a_{I-1} is that the invariant polynomials α_I from (43) are (nontrivial) objects from the local cohomology of the Koszul–Tate differential $H(\delta|\mathrm{d})$ at antighost number $I > 0$ and pure ghost number equal to 0, $\alpha_I \in H_I(\delta|\mathrm{d})$, i.e.

$$\delta\alpha_I = \partial_\mu j^\mu, \qquad \varepsilon(j^\mu) = 1, \qquad \mathrm{agh}(j^\mu) = I - 1, \qquad \mathrm{pgh}(j^\mu) = 0. \tag{44}$$

(We recall that the local cohomology $H(\delta|\mathrm{d})$ is completely trivial at both strictly positive antighost and pure ghost numbers—for instance, see [34], theorem 5.4 or [35]. In view of this result from now on it is understood that by $H_I(\delta|\mathrm{d})$ we mean the local cohomology of the Koszul–Tate differential at antighost I and at pure ghost number zero.) Consequently, we need to investigate some of the main properties of the local cohomology of the Koszul–Tate differential at strictly positive antighost numbers in order to completely determine the component a_I of highest antighost number in the first-order deformation. As the free model under study is a normal gauge theory of Cauchy order equal to 3, the general results from [34, 35] ensure that the local cohomology of the Koszul–Tate differential is trivial at antighost numbers strictly greater than its Cauchy order

$$H_k(\delta|\mathrm{d}) = 0, \qquad k > 3. \tag{45}$$

Moreover, if the invariant polynomial α_k, with $\mathrm{agh}(\alpha_k) = k \geqslant 3$, is trivial in $H_k(\delta|\mathrm{d})$, then it can be taken to be trivial also in $H_k^{\mathrm{inv}}(\delta|\mathrm{d})$:

$$\left(\alpha_k = \delta b_{k+1} + \partial_\mu \overset{(k)}{c}{}^{\mu},\ \mathrm{agh}(\alpha_k) = k \geqslant 3\right) \quad \Rightarrow \quad \alpha_k = \delta\beta_{k+1} + \partial_\mu \overset{(k)}{\gamma}{}^{\mu}, \tag{46}$$

where β_{k+1} and $\overset{(k)}{\gamma}{}^{\mu}$ are invariant polynomials. (An element of $H_k^{\mathrm{inv}}(\delta|\mathrm{d})$ is defined via an equation similar to (44), but with the corresponding current an invariant polynomial. The proof of (46) can be realized in the same manner like theorem 5 from [33], with the precaution to include in an appropriate manner the dependence on the vector field BRST sector.) Results (46) and (45) ensure that the entire local cohomology of the Koszul–Tate differential in the space of invariant polynomials is trivial in antighost numbers strictly greater than 3:

$$H_k^{\mathrm{inv}}(\delta|\mathrm{d}) = 0, \qquad k > 3. \tag{47}$$

Using definitions (18) and (20), we can organize the nontrivial representatives of $(H_k(\delta|\mathrm{d}))_{k\geqslant 2}$ and $(H_k^{\mathrm{inv}}(\delta|\mathrm{d}))_{k\geqslant 2}$ such as: (i) for $k > 3$ there are none (ii) for $k = 3$ they are linear combinations of the undifferentiated antifields $C^{*\mu\nu}$ with constant coefficients and (iii) for $k = 2$ they are written like linear combinations of the undifferentiated antifields $\eta^{*\mu\nu|\alpha}$ and η^* with constant coefficients. We have excluded from both $H(\delta|\mathrm{d})$ and $H^{\mathrm{inv}}(\delta|\mathrm{d})$ the nontrivial elements depending on the spacetime coordinates, as they would result in interactions with broken Poincaré invariance. In contrast to the groups $(H_k(\delta|\mathrm{d}))_{k\geqslant 2}$ and $\left(H_k^{\mathrm{inv}}(\delta|\mathrm{d})\right)_{k\geqslant 2}$, which are finite dimensional, the cohomology $H_1(\delta|\mathrm{d})$, that is related to global symmetries and ordinary conservation laws, is infinite dimensional since the theory is free. Fortunately, it will not be needed in the following.

The previous results on $H(\delta|\mathrm{d})$ and $H^{\mathrm{inv}}(\delta|\mathrm{d})$ at strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. As a consequence of result (47), we can eliminate all the terms with $k > 3$ from expansion (30) by adding only trivial pieces and thus work with $I \leqslant 3$.

4.2. Computation of the first-order deformation

Now, we have at hand all the necessary ingredients for computing the general form of the first-order deformation of the solution to the master equation. In the case $I = 3$, the nonintegrated density of the first-order deformation becomes

$$a = a_0 + a_1 + a_2 + a_3. \tag{48}$$

We can further decompose a in a natural manner as

$$a = a^{\mathrm{t}} + a^{\mathrm{t-A}} + a^{\mathrm{A}}, \tag{49}$$

where a^{t} contains only fields/ghosts/antifields from $t_{\mu\nu|\alpha\beta}$ sector, $a^{\mathrm{t-A}}$ describes the cross-interactions between the tensor field $t_{\mu\nu|\alpha\beta}$ and the vector field (so it effectively mixes both sectors) and a^{A} involves only the vector field sector. As it has been shown in [36] under the hypotheses of smoothness, locality, Lorentz covariance and Poincaré invariance of the deformations, combined with the requirement that the interacting Lagrangian is at most second-order derivative, to be maintained here as well, a^{t} satisfies an equation similar to (29) and has the expression

$$a^{\mathrm{t}} = c't \equiv t^{\mu\nu}{}_{|\mu\nu}, \tag{50}$$

with c' being an arbitrary, real constant. On the other hand, $a^{\mathrm{t-A}}$ and a^{A} involve different sorts of fields, so these components verify independently some equations similar to (29)

$$sa^{\mathrm{t-A}} = \partial_\mu m^{(\mathrm{t-A})\mu} \tag{51}$$

$$sa^{\mathrm{A}} = \partial_\mu m^{(\mathrm{A})\mu} \tag{52}$$

for some local m^μ s. In the following we analyse the general solutions to these equations.

The term $a^{\mathrm{t-A}}$ allows a decomposition similar to (48):

$$a^{\mathrm{t-A}} = a_0^{\mathrm{t-A}} + a_1^{\mathrm{t-A}} + a_2^{\mathrm{t-A}} + a_3^{\mathrm{t-A}}, \tag{53}$$

where the components of $a^{\mathrm{t-A}}$ are subject to the equations

$$\gamma a_3^{\mathrm{t-A}} = 0 \tag{54}$$

$$\delta a_I^{\mathrm{t-A}} + \gamma a_{I-1}^{\mathrm{t-A}} = \partial_\mu \overset{(I-1)}{m}{}^{(\mathrm{t-A})\mu}, \qquad I = 1, 2, 3. \tag{55}$$

In agreement with (43) and with the discussion made in the above regarding the nontrivial representatives of $H_3^{\mathrm{inv}}(\delta|\mathrm{d})$ (see the case (ii)) (54) possesses in $D \geqslant 5$ spacetime dimensions the solution

$$a_3^{\mathrm{t-A}} = a_3^{(1)\mathrm{t-A}} + a_3^{(2)\mathrm{t-A}}, \tag{56}$$

where

- for all $D \geqslant 5$

$$a_3^{(1)\mathrm{t-A}} = c_1 C^{*\mu\nu} C_{\mu\nu} \eta \tag{57}$$

- for $D = 5$

$$a_3^{(2)\mathrm{t-A}} = c_2 \varepsilon^{\mu\nu\lambda\beta\rho} C^*_{\mu\nu} (\partial_\lambda C_{\beta\rho}) \eta. \tag{58}$$

In relations (57)–(58) c_1 and c_2 are some arbitrary, real constants. Obviously, since components (57)–(58) are mutually independent, it follows that each of them must separately fulfil an equation of type (55) for $I = 3$:

$$\delta a_3^{(i)\mathrm{t-A}} = -\gamma a_2^{(i)\mathrm{t-A}} + \partial_\mu \overset{(2)}{m}{}^{(i)(\mathrm{t-A})\mu}, \qquad i = 1, 2. \tag{59}$$

By direct computation we obtain

$$\delta a_3^{(1)\mathrm{t-A}} = -\gamma \left[c_1 \eta^{*\mu\nu|\alpha} \left(\tfrac{3}{2} \eta_{\mu\nu|\alpha} \eta - C_{\mu\nu} A_\alpha\right)\right] + \partial_\mu u^\mu + \tfrac{3}{2} c_1 \eta^{*\mu\nu|\alpha} \partial_{[\mu} C_{\nu]\alpha} \eta. \tag{60}$$

Thus, $a_3^{(1)\mathrm{t-A}}$ produces a consistent $a_2^{(1)\mathrm{t-A}}$ as solution to (59) for $i = 1$ if and only if the term $(3/2) c_1 \eta^{*\mu\nu|\alpha} \partial_{[\mu} C_{\nu]\alpha} \eta$, which is a nontrivial representative of $H(\gamma)$, is written in a γ-exact modulo d form. This takes place if and only if

$$c_1 = 0. \tag{61}$$

Related to $a_3^{(2)\mathrm{t-A}}$, by applying δ on (58) we find via (59) for $i = 2$ that $a_2^{(2)\mathrm{t-A}}$ reads as

$$a_2^{(2)\mathrm{t-A}} = 3 c_2 \varepsilon^{\mu\nu\lambda\beta\rho} \eta^{*}_{\mu\nu|}{}^{\alpha} \left[\left(\tfrac{5}{12} \partial_\lambda \eta_{\beta\rho|\alpha} + \tfrac{1}{6} \partial_\beta \eta_{\rho\alpha|\lambda}\right) \eta - (\partial_\lambda C_{\beta\rho}) A_\alpha\right]. \tag{62}$$

By means of (62) we deduce

$$\delta a_2^{(2)\mathrm{t-A}} = \partial_\mu v^\mu - \gamma\big[c_2\varepsilon^{\mu\nu\lambda\beta\rho}t^{*\tau\alpha|}_{\mu\nu}(3(\partial_\lambda t_{\tau\alpha|\beta\rho})\eta + (5\partial_\lambda\eta_{\beta\rho|\tau} + 2\partial_\beta\eta_{\rho\tau|\lambda})A_\alpha)\big] - 6c_2\varepsilon^{\mu\nu\lambda\beta\rho}t^{*\tau\alpha|}_{\mu\nu}(\partial_\lambda C_{\beta\rho})F_{\tau\alpha}. \tag{63}$$

Comparing (63) with (55), for $I = 2$, we can state that $a_2^{(2)\mathrm{t-A}}$ provides a consistent $a_1^{(2)\mathrm{t-A}}$ if and only if the term $6c_2\varepsilon^{\mu\nu\lambda\beta\rho}t^{*\tau\alpha|}_{\mu\nu}(\partial_\lambda C_{\beta\rho})F_{\tau\alpha}$, which is again a nontrivial representative of $H(\gamma)$, is γ -exact modulo d. This holds if and only if

$$c_2 = 0. \tag{64}$$

Results (61) and (64) show that

$$a_3^{\mathrm{t-A}} = 0. \tag{65}$$

Accordingly, $a^{\mathrm{t-A}}$ can stop earliest at antighost number 2:

$$a^{\mathrm{t-A}} = a_0^{\mathrm{t-A}} + a_1^{\mathrm{t-A}} + a_2^{\mathrm{t-A}}, \tag{66}$$

with $a_2^{\mathrm{t-A}}$ being a solution to the equation $\gamma a_2^{\mathrm{t-A}} = 0$. Looking at (43) for $I = 2$, using the previous results on the nontrivial representatives of $H_2^{\mathrm{inv}}(\delta|\mathrm{d})$ (see the case (iii) in the above), and requiring that $a_2^{\mathrm{t-A}}$ effectively describes cross-couplings, we get (up to trivial, γ-exact contributions) that

$$a_2^{\mathrm{t-A}} = \eta^*(\lambda^{\mu\nu}C_{\mu\nu} + \bar\lambda^{\mu\nu\alpha}\partial_{[\mu}C_{\nu]\alpha}), \tag{67}$$

where $\lambda^{\mu\nu}$ and $\bar\lambda^{\mu\nu\alpha}$ are some non-derivative, real constant tensors, invariant under the Lorentz group, with $\lambda^{\mu\nu}$ and $\bar\lambda^{\mu\nu\alpha}$ antisymmetric in λ and μ. Since in $D \geqslant 5$ there are no such constant tensors, we must set $\lambda^{\mu\nu} = 0$ and $\bar\lambda^{\mu\nu\alpha} = 0$, so we have that

$$a_2^{\mathrm{t-A}} = 0. \tag{68}$$

In this way we infer that $a^{\mathrm{t-A}}$ actually stops at antighost number 1:

$$a^{\mathrm{t-A}} = a_0^{\mathrm{t-A}} + a_1^{\mathrm{t-A}}, \tag{69}$$

with $a_1^{\mathrm{t-A}}$ being a solution to the equation $\gamma a_1^{\mathrm{t-A}} = 0$. Since $a_1^{\mathrm{t-A}}$ is linear in the antifields of the original fields, we can write

$$a_1^{\mathrm{t-A}} = (t^*_{\mu\nu|\alpha\beta}\Delta^{\mu\nu|\alpha\beta} + A^*_\mu\Delta^\mu)\eta, \tag{70}$$

where $\Delta^{\mu\nu|\alpha\beta}$ and Δ^μ are γ -invariant objects (with both the antighost number and the pure ghost number equal to 0), the former quantities displaying the mixed symmetry of the tensor $t^{\mu\nu|\alpha\beta}$. From (43) at antighost number 0 we observe that $\Delta^{\mu\nu|\alpha\beta}$ and Δ^μ depend in general on $F^{\mu\nu}$, $F^{\mu\nu\lambda|\alpha\beta\gamma}$ and their derivatives. Moreover, the requirement that the second term on the right-hand side of (70) produces cross-interactions implies that Δ^μ must involve $F^{\mu\nu\lambda|\alpha\beta\gamma}$ (and possibly its derivatives). In order to construct $a_0^{\mathrm{t-A}}$ as solution to the equation

$$\delta a_1^{\mathrm{t-A}} + \gamma a_0^{\mathrm{t-A}} = \partial_\mu \overset{(0)}{m}{}^{(\mathrm{t-A})\mu}, \tag{71}$$

we invoke the hypothesis on the maximum derivative order of $a_0^{\mathrm{t-A}}$ being equal to 2. As both $\delta t^*_{\mu\nu|\alpha\beta}$ and δA^*_μ contain exactly two derivatives, it follows that each of $\Delta^{\mu\nu|\alpha\beta}$ and Δ^μ are allowed to include at most one derivative (we remind that $\partial_\mu\eta = \gamma A_\mu$) and therefore we have that

$$\Delta^{\mu\nu|\alpha\beta} = \Delta_0^{\mu\nu|\alpha\beta} + \Delta_1^{\mu\nu|\alpha\beta}, \qquad \Delta^\mu = \Delta_0^\mu + \Delta_1^\mu, \tag{72}$$

where Δ_0s contain no derivatives and Δ_1s include just one derivative. Therefore, both Δ_0s must be constant, while Δ_1s must depend linearly on $F_{\mu\nu}$. From covariance arguments in $D \geqslant 5$ we have that the only possible choice of these quantities is

$$\Delta_0^{\mu\nu|\alpha\beta} = \tfrac{1}{2}k_1(\sigma^{\mu\alpha}\sigma^{\nu\beta} - \sigma^{\mu\beta}\sigma^{\nu\alpha}), \qquad \Delta_0^\mu = 0, \tag{73}$$

and

$$\Delta_1^{\mu\nu|\alpha\beta} = k_2 \varepsilon^{\mu\nu\alpha\beta\lambda\rho} F_{\lambda\rho}, \qquad \Delta_1^{\mu} = 0, \tag{74}$$

with k_1 and k_2 being arbitrary, real constants. Solution (74) 'lives' in $D = 6$, but it brings no contribution to $a_1^{\mathrm{t-A}}$ as $\varepsilon^{\mu\nu\alpha\beta\lambda\rho} t^*_{\mu\nu|\alpha\beta} \equiv 0$. Substituting (72)–(74) in (70) we find that

$$a_1^{\mathrm{t-A}} = k_1 t^* \eta. \tag{75}$$

By applying δ on (75) we deduce

$$\delta a_1^{\mathrm{t-A}} = \frac{k_1}{4}(4-D)(3-D)\left(\partial^\mu \partial^\rho t_{\mu\rho} - \frac{1}{2}\Box t\right)\eta. \tag{76}$$

In order to analyse the solution to (71), we assume that $a_1^{\mathrm{t-A}}$ of the form (75) generates a consistent $a_0^{\mathrm{t-A}}$. From (76) it follows that the corresponding $a_0^{\mathrm{t-A}}$ is linear in both $t_{\mu\nu|\alpha\beta}$ and A_μ and contains precisely one spacetime derivative. Then, up to an irrelevant divergence, $a_0^{\mathrm{t-A}}$ reads as

$$a_0^{\mathrm{t-A}} = A_\mu m^\mu(\partial t), \tag{77}$$

where $m^\mu(\partial t)$ is linear in the first-order derivatives of $t_{\mu\nu|\alpha\beta}$. It is simple to see that the most general form of $m^\mu(\partial t)$ can be represented like

$$m^\mu(\partial t) = c_3 \partial^\mu t + c_4 \partial_\rho t^{\mu\rho}, \tag{78}$$

with c_3 and c_4 being arbitrary, real constants, such that

$$a_0^{\mathrm{t-A}} = A_\mu (c_3 \partial^\mu t + c_4 \partial_\rho t^{\mu\rho}). \tag{79}$$

By direct computation, from (76) and (79) we obtain

$$\begin{aligned}\delta a_1^{\mathrm{t-A}} + \gamma a_0^{\mathrm{t-A}} = {} & \partial_\mu \left[\frac{k_1}{4}(4-D)(3-D)\left(\partial_\rho t^{\mu\rho} - \frac{1}{2}\partial^\mu t\right)\eta + (4c_3 + c_4) A^\mu \partial^\rho \eta_{\rho\beta|}{}^{\beta}\right. \\ & \left. + c_4 A^\alpha \left(\partial^\mu \eta_{\alpha\beta|}{}^{\beta} - \partial^\beta \eta_{\alpha\beta|}{}^{\mu}\right)\right] + \left(c_3 + \frac{k_1}{8}(4-D)(3-D)\right)(\partial^\mu t)\partial_\mu \eta \\ & + \left(c_4 - \frac{k_1}{4}(4-D)(3-D)\right)(\partial_\rho t^{\mu\rho})\partial_\mu \eta - (4c_3 + c_4)(\partial_\mu A^\mu)\partial^\rho \eta_{\rho\beta|}{}^{\beta} \\ & - c_4 (\partial_\mu A^\alpha)\left(\partial^\mu \eta_{\alpha\beta|}{}^{\beta} - \partial^\beta \eta_{\alpha\beta|}{}^{\mu}\right).\end{aligned} \tag{80}$$

The right-hand side of (80) reduces to a total derivative (as it is required by (71)) if and only if the constants k_1, c_3 and c_4 satisfy the equations

$$c_3 + \frac{k_1}{8}(4-D)(3-D) = 0, \qquad 4c_3 + c_4 = 0 \tag{81}$$

$$c_4 - \frac{k_1}{4}(4-D)(3-D) = 0, \qquad c_4 = 0 \tag{82}$$

allowing only the vanishing solution

$$k_1 = c_3 = c_4 = 0. \tag{83}$$

As a consequence, we find that

$$a_1^{\mathrm{t-A}} = 0, \tag{84}$$

so $a^{\mathrm{t-A}}$ actually reduces to its component of antighost number zero,

$$a^{\mathrm{t-A}} = a_0^{\mathrm{t-A}}, \tag{85}$$

which is subject to the 'homogeneous' equation

$$\gamma a_0^{\mathrm{t-A}} = \partial_\mu \overset{(0)}{m}{}^{(\mathrm{t-A})\mu}. \tag{86}$$

There are two main types of solutions to this equation. The first type, to be denoted by $a_0^{\prime\mathrm{t-A}}$, corresponds to $\overset{(0)}{m}{}^{(\mathrm{t-A})\mu} = 0$ and is given by gauge-invariant, nonintegrated densities constructed out of the original fields and their spacetime derivatives, which, according to (43), are of the form

$$a_0^{\prime\mathrm{t-A}} = a_0^{\prime\mathrm{t-A}}([F_{\mu\nu}], [F_{\mu\nu\lambda|\alpha\beta\gamma}]) \tag{87}$$

up to the condition that they effectively describe cross-couplings between the two types of fields and cannot be written in a divergence-like form. Such a solution would produce vertices with more than two derivatives of the fields and must be excluded since this disagrees with the hypothesis on the maximum derivative order:

$$a_0^{\prime\mathrm{t-A}} = 0. \tag{88}$$

(If we however relax the derivative-order condition, then we can find nonvanishing solutions of type (87). An example of a possible solution is represented by the cubic vertex $a_0^{\prime\mathrm{t-A}} = F_{\mu\nu\lambda|\alpha\beta\gamma} F^{\mu\nu} F^{\alpha\beta} \sigma^{\lambda\gamma}$.)

The second kind of solutions, to be denoted by $a_0^{\prime\prime\mathrm{t-A}}$, is associated with $\overset{(0)}{m}{}^{(\mathrm{t-A})\mu} \neq 0$ in (86), being understood that we discard the divergence-like quantities and maintain the condition on the maximum derivative order. In order to solve the equation

$$\gamma a_0^{\prime\prime\mathrm{t-A}} = \partial_\mu \overset{(0)}{m}{}^{(\mathrm{t-A})\mu}, \tag{89}$$

we start from the requirement that $a_0^{\prime\prime\mathrm{t-A}}$ may contain at most two derivatives. Then, $a_0^{\prime\prime\mathrm{t-A}}$ can be decomposed as

$$a_0^{\prime\prime\mathrm{t-A}} = \omega_0 + \omega_1 + \omega_2, \tag{90}$$

where $(\omega_i)_{i=\overline{0,2}}$ contains i derivatives. Due to the different number of derivatives in the components ω_0, ω_1, and ω_2, (89) leads to three independent equations:

$$\gamma \omega_k = \partial_\mu j_k^\mu, \qquad k = 0, 1, 2. \tag{91}$$

For $k = 0$, (91) implies the necessary conditions $\partial_\mu(\partial\omega_0/\partial t_{\mu\nu|\alpha\beta}) = 0$ and $\partial_\mu(\partial\omega_0/\partial A_\mu) = 0$, whose solutions read as $\partial\omega_0/\partial t_{\mu\nu|\alpha\beta} = k^{\mu\nu|\alpha\beta}$ and $\partial\omega_0/\partial A_\mu = k^\mu$, where $k^{\mu\nu|\alpha\beta}$ and k^μ are arbitrary, real constants. The last solutions provide $\omega_0 = k^{\mu\nu|\alpha\beta} t_{\mu\nu|\alpha\beta} + k^\mu A_\mu$, so it does not describe cross-interactions between $t_{\mu\nu|\alpha\beta}$ and A_μ and can be made to vanish, $\omega_0 = 0$. For $k = 1$, (91) requires that

$$\partial_\mu \frac{\delta\omega_1}{\delta t_{\mu\nu|\alpha\beta}} = 0, \qquad \partial_\mu \frac{\delta\omega_1}{\delta A_\mu} = 0, \tag{92}$$

whose solutions are of the type

$$\frac{\delta\omega_1}{\delta t_{\mu\nu|\alpha\beta}} = 0, \qquad \frac{\delta\omega_1}{\delta A_\mu} = \partial_\nu B^{\mu\nu}, \tag{93}$$

where $\delta\omega_1/\delta t_{\mu\nu|\alpha\beta}$ and $\delta\omega_1/\delta A_\mu$ denote the variational derivatives of ω_1. In the above the antisymmetric functions $B^{\mu\nu}$ have no derivatives. (In general, the solution to the equation $\partial_\mu(\delta\alpha/\delta t_{\mu\nu|\alpha\beta}) = 0$ has the form $\delta\alpha/\delta t_{\mu\nu|\alpha\beta} = \partial_\lambda\partial_\rho M^{\mu\nu\lambda|\alpha\beta\rho} + m^{\mu\nu|\alpha\beta}$, where the functions $M^{\mu\nu\lambda|\alpha\beta\rho}$ display the mixed symmetry of the curvature tensor and $m^{\mu\nu|\alpha\beta}$ are some non-derivative constants with the mixed symmetry $(2, 2)$. If we ask that α comprises one spacetime derivative, then we must set $M^{\mu\nu\lambda|\alpha\beta\rho} = 0$ and $m^{\mu\nu|\alpha\beta} = 0$, which justifies the former solution from (93).) Using (93), we conclude that, up to an irrelevant divergence, ω_1 is a functions of A_μ with precisely one derivative. Such an ω_1 does not provide cross-interactions between $t_{\mu\nu|\alpha\beta}$ and A_μ, so we can take $\omega_1 = 0$.

In the following we consider (91) for $k = 2$, which gives the necessary conditions

$$\partial_\mu \frac{\delta\omega_2}{\delta t_{\mu\nu|\alpha\beta}} = 0, \qquad \partial_\mu \frac{\delta\omega_2}{\delta A_\mu} = 0, \tag{94}$$

with the solutions

$$\frac{\delta\omega_2}{\delta t_{\mu\nu|\alpha\beta}} = \partial_\lambda\partial_\rho U^{\mu\nu\lambda|\alpha\beta\rho}, \qquad \frac{\delta\omega_2}{\delta A_\mu} = \partial_\nu \Phi^{\mu\nu}. \tag{95}$$

Let N be a derivation in the algebra of the fields and of their derivatives, which counts the powers of the fields and their derivatives, defined by

$$N = \sum_{k\geqslant 0}\left(\left(\partial_{\mu_1}\ldots\partial_{\mu_k} t_{\mu\nu|\alpha\beta}\right)\frac{\partial}{\partial\left(\partial_{\mu_1}\ldots\partial_{\mu_k} t_{\mu\nu|\alpha\beta}\right)} + \left(\partial_{\mu_1}\ldots\partial_{\mu_k} A_\mu\right)\frac{\partial}{\partial\left(\partial_{\mu_1}\ldots\partial_{\mu_k} A_\mu\right)}\right). \tag{96}$$

Then, it is easy to see that for every nonintegrated density χ, we have that

$$N\chi = t_{\mu\nu|\alpha\beta}\frac{\delta\chi}{\delta t_{\mu\nu|\alpha\beta}} + A_\mu \frac{\delta\chi}{\delta A_\mu} + \partial_\mu s^\mu. \tag{97}$$

If $\chi^{(l)}$ is a homogeneous polynomial of order $l > 0$ in the fields $\{t_{\mu\nu|\alpha\beta}, A_\mu\}$ and their derivatives, then $N\chi^{(l)} = l\chi^{(l)}$. On account of (95) and (97), we find that

$$N\omega_2 = \tfrac{1}{9} F_{\mu\nu\lambda|\alpha\beta\rho} U^{\mu\nu\lambda|\alpha\beta\rho} + \tfrac{1}{2} F_{\mu\nu}\Phi^{\mu\nu} + \partial_\mu v^\mu. \tag{98}$$

We expand ω_2 as

$$\omega_2 = \sum_{l>0} \omega_2^{(l)}, \tag{99}$$

where $N\omega_2^{(l)} = l\omega_2^{(l)}$, such that

$$N\omega_2 = \sum_{l>0} l\omega_2^{(l)}. \tag{100}$$

Comparing (98) with (100), we reach the conclusion that decomposition (99) induces a similar decomposition with respect to $U^{\mu\nu\lambda|\alpha\beta\rho}$ and $\Phi^{\mu\nu}$, i.e.

$$U^{\mu\nu\lambda|\alpha\beta\rho} = \sum_{l>0} U^{\mu\nu\lambda|\alpha\beta\rho}_{(l-1)}, \qquad \Phi^{\mu\nu} = \sum_{l>0}\Phi^{\mu\nu}_{(l-1)}. \tag{101}$$

Substituting (101) into (98) and comparing the resulting expression with (100), we obtain that

$$\omega_2^{(l)} = \frac{1}{9l} F_{\mu\nu\lambda|\alpha\beta\rho} U^{\mu\nu\lambda|\alpha\beta\rho}_{(l-1)} + \frac{1}{2l} F_{\mu\nu}\Phi^{\mu\nu}_{(l-1)} + \partial_\mu \bar{v}^\mu_{(l)}. \tag{102}$$

Introducing (102) in (99), we arrive at

$$\omega_2 = F_{\mu\nu\lambda|\alpha\beta\rho}\bar{U}^{\mu\nu\lambda|\alpha\beta\rho} + F_{\mu\nu}\bar{\Phi}^{\mu\nu} + \partial_\mu \bar{v}^\mu, \tag{103}$$

where

$$\bar{U}^{\mu\nu\lambda|\alpha\beta\rho} = \sum_{l>0}\frac{1}{9l} U^{\mu\nu\lambda|\alpha\beta\rho}_{(l-1)}, \qquad \bar{\Phi}^{\mu\nu} = \sum_{l>0}\frac{1}{2l}\Phi^{\mu\nu}_{(l-1)}. \tag{104}$$

By applying γ on relation (103), after long and tedious computation we infer that a necessary condition for the existence of solutions to the equation $\gamma\omega_2 = \partial_\mu j_2^\mu$ is that the functions $\bar{U}^{\mu\nu\lambda|\alpha\beta\rho}$ and $\bar{\Phi}^{\mu\nu}$ have the expressions

$$\bar{U}^{\mu\nu\lambda|\alpha\beta\rho} = C^{\mu\nu\lambda|\alpha\beta\rho;\sigma} A_\sigma, \qquad \bar{\Phi}^{\mu\nu} = \bar{k}^{\mu\nu\rho;\alpha\beta|\sigma\lambda}\partial_\rho t_{\alpha\beta|\sigma\lambda}, \tag{105}$$

where $C^{\mu\nu\lambda|\alpha\beta\rho;\sigma}$ and $\bar{k}^{\mu\nu\rho;\alpha\beta|\sigma\lambda}$ are non-derivative, real constants. The former constants exhibit the mixed symmetry (3, 3) in the indices $\mu\nu\lambda|\alpha\beta\rho$ and are separately antisymmetric in $\{\alpha, \beta, \rho, \sigma\}$. The quantities $\bar{k}^{\mu\nu\rho;\alpha\beta|\sigma\lambda}$ are antisymmetric in the indices $\{\mu, \nu, \rho\}$ and display the mixed symmetry (2, 2) with respect to $\alpha\beta|\sigma\lambda$. Substituting (105) in (103) we get that

$$\omega_2 = C^{\mu\nu\lambda|\alpha\beta\rho;\sigma} F_{\mu\nu\lambda|\alpha\beta\rho} A_\sigma + \partial_\rho(F_{\mu\nu}\bar{k}^{\mu\nu\rho;\alpha\beta|\sigma\lambda} t_{\alpha\beta|\sigma\lambda} + \bar{v}^\rho). \tag{106}$$

As a consequence, the existence of a nontrivial ω_2 is conditioned by the existence of some pure constants $C^{\mu\nu\lambda|\alpha\beta\rho;\sigma}$ that must simultaneously display the mixed symmetry (3, 3) in their first six indices and be antisymmetric in the indices $\{\alpha, \beta, \rho, \sigma\}$. Because of the odd number of indices in $C^{\mu\nu\lambda|\alpha\beta\rho;\sigma}$, these constants can only be constructed from the flat metric $\sigma^{\mu\nu}$ and Levi-Civita symbols $\varepsilon^{\mu_1\cdots\mu_j}$. Due to the identity $F_{[\mu\nu\lambda|\alpha]\beta\rho} \equiv 0$, the Levi-Civita symbols can be contracted with $F_{\mu\nu\lambda|\alpha\beta\rho}$ on at most three indices. On the other hand, the restriction $D \geqslant 5$ on the spacetime dimension requires the Levi-Civita symbols with at least five indices, so $\varepsilon^{\mu_1\cdots\mu_j}$ will contract with $F_{\mu\nu\lambda|\alpha\beta\rho}$ on at least four indices, such that the corresponding ω_2 will vanish identically. In consequence, we can take

$$C^{\mu\nu\lambda|\alpha\beta\rho;\sigma} = 0,$$

which further leads to

$$a_0^{\prime\prime\mathrm{t-A}} = 0. \tag{107}$$

Relations (88) and (107) show that

$$a_0^{\mathrm{t-A}} = 0. \tag{108}$$

By means of results (53), (65), (68), (84) and (108) we arrive at

$$a^{\mathrm{t-A}} = 0. \tag{109}$$

Finally, we focus on the solutions to (52). It is easy to see that a^{A} can only reduce to its component of antighost number zero

$$a^{\mathrm{A}} = a_0^{\mathrm{A}}([A_\mu]), \tag{110}$$

which is solution to the equation $sa^{\mathrm{A}} \equiv \gamma a_0^{\mathrm{A}} = \partial_\mu m_0^{(\mathrm{A})\mu}$. It comes from $a_1^{\mathrm{A}} = 0$ and does not deform the gauge transformations, but merely modifies the vector field action. The condition that a_0^{A} is of maximum derivative order equal to 2 is translated into

$$a_0^{\mathrm{A}} = c''\varepsilon^{\mu\nu\lambda\beta\rho} A_\mu F_{\nu\lambda} F_{\beta\rho} \tag{111}$$

for $D = 5$, with c'' an arbitrary, real constant. Putting together the results deduced so far, we obtained that the first-order deformation of the solution to the master equation for theory (1) has the expression

$$a = c't + c''\varepsilon^{\mu\nu\lambda\beta\rho} A_\mu F_{\nu\lambda} F_{\beta\rho}. \tag{112}$$

4.3. Higher order deformations

Taking into account (28), etc, we get that the first-order deformation (112) is consistent to all orders in the coupling constant. Indeed, as $(S_1, S_1) = 0$, it follows that (28), which describes the second-order deformation, is satisfied with the choice

$$S_2 = 0, \tag{113}$$

while the remaining higher order equations are fulfilled for

$$S_3 = S_4 = \cdots = 0. \tag{114}$$

The fact that $a^{\mathrm{t-A}} = 0$ shows there are no consistent cross-couplings between the massless tensor field $t_{\mu\nu|\alpha\beta}$ and the vector field A_μ complying with all the hypotheses used in this paper.

5. Conclusion

To conclude, in this paper we have investigated the couplings between the massless tensor field with the mixed symmetry of the Riemann tensor and the massless vector field by using the powerful setting based on local BRST cohomology. Under the assumptions on smoothness, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the requirement that the interacting Lagrangian is at most second-order derivative, we have proved that there are no consistent cross-interactions between such fields. Our approach opens the perspective of investigating the interactions between the tensor field $t_{\mu\nu|\alpha\beta}$ and one p-form ($p > 1$) or, more general, between a tensor field with the mixed symmetry (k, k) and a p-form. These problems are under consideration.

Acknowledgments

The authors are partially supported by the European Commission FP6 program MRTN-CT-2004-005104 and by the type A grant 304/2004 with the Romanian National Council for Academic Scientific Research and the Romanian Ministry of Education and Research.

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